

## MICROWAVE PERMEABILITY TENSOR OF PARTIALLY MAGNETIZED FERRITES

MITSURU IGARASHI  
 Dept. of Electrical Engineering  
 OYAMA TECHNICAL COLLEGE,  
 OYAMA 323 JAPAN

YOSHIYUKI NAITO  
 Dept. of Physical Electronics  
 TOKYO INSTITUTE OF TECHNOLOGY,  
 TOKYO 152 JAPAN.

### Abstract

The microwave permeability tensor of partially magnetized ferrites is formulated. The theory clarifies that the remarkable low-field losses disappear if  $\omega_M/\omega \leq \sqrt{5}/3$ . The theory also is in good agreement with experimental results.

### I. Introduction

In the actual applications of ferrites, partially magnetized ferrites are very important for the devices such as circulators operating below resonance, latching phase shifters and so on. The researches on the tensor permeability of partially magnetized ferrites, or more especially magnetic losses are also of great importance. Polder's theory (1) of tensor permeability of magnetized ferromagnetic materials has been used under the condition that the materials are magnetically saturated. Rado (2) obtained the off-diagonal element  $\kappa$  in nonsaturated ferromagnetic materials under some conditions and assumptions. Later, Schloemann (3, 4) presented the formula about the (isotropic) permeability in the completely demagnetized state. Schloemann's formula cannot be used, when there exists the externally applied dc magnetic field of arbitrary strength. Polder and Smit's analysis (5) is convenient to clarify the regions of losses due to spin resonances qualitatively but helpless to get the magnitude of losses quantitatively.

As far as the authors know, there have been no reports in which the theoretical formula of tensor permeability in the partially magnetized state are discussed successfully. The theoretical formula in this report can be applied throughout the partially magnetized region and may extend over Schloemann's theory. The materials have not remarkable low-field losses if  $\omega_M/\omega \leq \sqrt{5}/3$ , where  $\omega_M$  and  $\omega$  are angular saturation magnetization frequency and angular RF frequency, respectively.

### II. Theoretical Analysis

We will discuss here in the case of the idealized domain model. That is, the effects due to the exchange field is assumed to be negligible and an external dc magnetic field is applied along the positive z-axis. Consider each one of two or more domains magnetized parallel to the z-axis or antiparallel to the z-axis. (a) The parallel domains present a scalar permeability  $\mu_+$  for the clockwise rotating field and a scalar permeability  $\mu_-$  for the counter-clockwise one. The antiparallel domains, on the other hand, present  $\mu_-$  for the clockwise rotating field and  $\mu_+$  for the counter-clockwise one. Let's assume that the resultant permeability for the clockwise (or counter-clockwise) rotating field can be expressed by means of the weighted geometric average of  $\mu_+$  and  $\mu_-$ . Next, we use the usual method of geometric average and superposition in order to get the diagonal element of resultant tensor permeability for the linearly polarized RF

field. Provided that the domain walls are mostly  $180^\circ$  walls and also arranged in the direction of the x-axis (or y-axis), the diagonal element obtained in the case (a) shows the average of the x-direction (or y-direction). (b) Provided that every magnetic wall is a  $180^\circ$  wall and also arranged in the z-direction. So long as the internal magnetic field  $H_i$  is equal to zero, the z-component of the magnetization does not appear and the material is not magnetized at all. Thus for this case the diagonal element is always equal to unity.

The weight (volume ratio) of the case (a) and that of the case (b) are  $2/3$  and  $1/3$ , respectively. Therefore for partially magnetized materials with random domain orientation, the diagonal element of the effective tensor permeability of two or more domains,  $\langle\langle \mu \rangle\rangle$ , is given by

$$\langle\langle \mu \rangle\rangle = \frac{2}{3} \{ \sqrt{\mu_+ \mu_-} (1 - \langle \alpha_3 \rangle^2) + \mu \langle \alpha_3 \rangle^2 \} + \frac{1}{3} \quad (1)$$

$$\text{where } \mu_{\pm} = \mu \mp \kappa = 1 + \frac{\omega_M}{\mp \omega + \omega_e + j\omega a}$$

$$\omega_e = -\gamma H_{\text{eff}} \approx -\gamma H_a$$

$$H_a = \text{anisotropy field}$$

and  $\langle \alpha_3 \rangle$  is the ratio of magnetization  $M$  to the saturation magnetization  $M_s$ . This ratio could be given as an approximated expression:

$$\langle \alpha_3 \rangle \approx \sin(bH_N) \quad (2)$$

$$\text{where } H_N = \frac{H}{N_z M_s}$$

$N_z$  = demagnetizing factor of z-direction.

Here,  $b$  is so determined that effective tensor permeability, i. e., the real part and the imaginary part of eq. (1) coincide with the experimental values of them at the internal dc magnetic field  $H_i \rightarrow 0$ . Also  $\omega_e/\omega$  will be, if necessary, evaluated from the measured value of  $\langle\langle \mu \rangle\rangle$  at  $H_N = 0$ . The determination of parameters is omitted here (the results will be shown in FIGURES).

We obtained the loss term at  $H_N = 0$  and at a point where  $H_i \rightarrow 0$  after complicated

calculations. We also determined the condition and the parameter  $b$  where remarkable low-field losses disappear. The results obtained from this theory are as follows:

$$\text{condition: } \frac{\omega_M}{\omega} \leq \frac{\sqrt{5}}{3} \quad (3)$$

$$\text{parameter: } b = 0 \text{ for } \frac{\omega_M}{\omega} = \frac{\sqrt{5}}{3}. \quad (4)$$

Therefore the low-field loss should be constant: for  $\omega_M/\omega = \sqrt{5}/3$

$$\ll \mu \gg = \frac{\sqrt{5}}{3} \cdot \alpha (\text{constant}), \quad (5)$$

Eq. (3) gives the condition of the low magnetic loss for partially magnetized materials and eq. (5). Also that the magnetic losses are constant approximately. Either Polder and Smit's theory or Schloemann's theory shows only  $\omega_M/\omega \ll 1$  (or  $\omega_M/\omega < 1$ ) as a condition for low losses but never clarifies the condition and the results as mentioned above. Experimental results (6-9) are, for the first time, clarified in this theory.

### III. Experimental Results

The authors have measured the tensor permeability of a number of samples to obtain the generalized representation of microwave losses for partially magnetized ferrites (8, 9). The experimental results are shown in the following figures. The quantitative results of the samples are also shown in Table 1.

(1) Real part of  $\ll \mu \gg$  In Fig. 1, the real part of  $\ll \mu \gg$  obtained from eq. (1) and (2) is compared with the results measured by Green et al. (7).

(2) Loss term of  $\ll \mu \gg$  In Fig. 2 and 3, the loss term obtained from eq. (1) and (2) is compared with the measured values, where  $\mu_N''$  means the loss term normalized by the value of loss term at  $H_N = 0$ :  $\mu_N'' = \ll \mu \gg'' / \mu_0''$ . As obvious from Fig. 1, 2 and 3, the theory is in good agreement with the experimental results.

(3) Low-field losses As discussed above, the remarkable low-field losses disappear under the condition that  $\omega_M/\omega \leq \sqrt{5}/3$ . In the experiments, the remarkable low-field losses disappear without exception in the case where the samples satisfy the condition:  $\omega_M/\omega \leq 0.72$  (we had no sample of which  $\omega_M/\omega = \sqrt{5}/3 \approx 0.745$ ). As obvious from Fig. 4, the graph of  $\mu_N''$  is almost constant and shown that there is no remarkable low-field losses.

### IV. Conclusion

The microwave permeability tensor of partially magnetized ferrites is, for the first time, formalized. The theory is in good agreement with the experimental results, practical use and useful. It is clarified in theory and experiment that the remarkable low-field losses disappear if  $\omega_M/\omega \leq \sqrt{5}/3$ .

### Acknowledgements

It is a great delight to thank Dr. E. Schloemann, Dr. J. J. Green, Dr. D. Massé of Raytheon Research Division, U. S. A., and Prof. C. E. Patton of Colorado State University, U. S. A., for stimulating discussions, and to Mr. R. Ishii of Fuji Denki Kagaku Co., Ltd., Japan, and Mr. H. Kurebayashi of Mitsubishi Denki Co., Ltd., Japan, for facilities for experiments.

### References

- (1) D. Polder, "On the theory of ferromagnetic resonance," *Phil. Mag.*, vol. 40, pp. 99-115 1949.
- (2) G. T. Rado, "Theory of the microwave permeability tensor and Faraday effect in nonsaturated ferromagnetic materials", *Phys. Rev.*, vol. 89, pp. 529, 1953.
- (3) E. Schloemann, "Microwave behavior of partially magnetized ferrites," *J. Appl. Phys.*, vol. 41, pp. 204-214, 1970.
- (4) E. Schloemann, "Behavior of ferrites in the microwave frequency range," *J. Phys.* 32, pp. C1-443-451, 1971.
- (5) D. Polder and J. Smit, "Resonance phenomena in ferrites," *Rev. Mod. Phys.* vol. 25, pp. 89-90, 1953.
- (6) R. Roveda et. al., "Dissipative parameters in ferrites and insertion losses in waveguide Y-circulators below resonance," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 89-96, 1972.
- (7) J. J. Green and F. Sandy, "Microwave characterization of partially magnetized ferrites," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-22, pp. 641-645, June 1974.
- (8) M. Igarashi, Doctoral thesis (Chap. 2 and 3), Tokyo Institute of Technology, Tokyo, Japan, Mar. 1975.
- (9) M. Igarashi and Y. Naito, "Microwave losses of partially magnetized ferrites," presented at the Inst. Elec. Eng. Japan Conv. Mar. 1975, Paper S8-8.

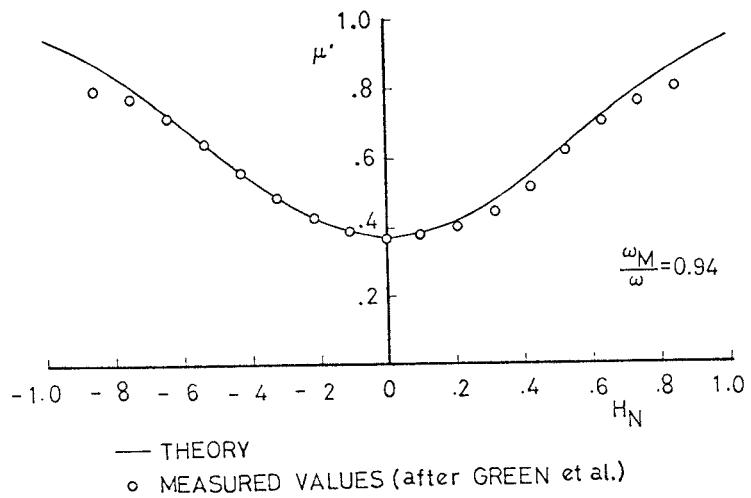


Fig. 1  $\mu'$  versus  $H_N$  on TT1-390 at 5.5 GHz, where  $\mu'(H_i \rightarrow 0) = 0.95$ ,  $\omega_e/\omega = 0.058$  and  $b = 80^\circ$ .

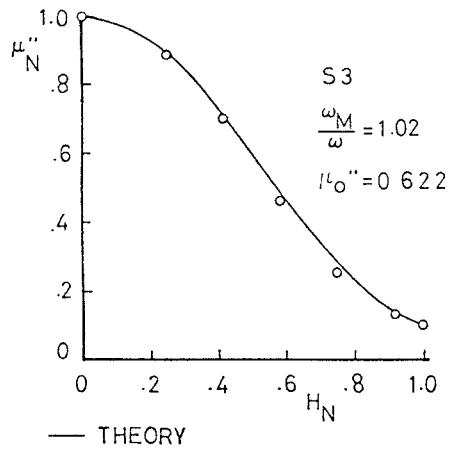


Fig. 2  $\mu_N''$  versus  $H_N$  on S3 at 3.8 GHz, where  $\mu_0'' = 0.622$ ,  $\omega_e/\omega = 0.376$  and  $b = 80^\circ$ .

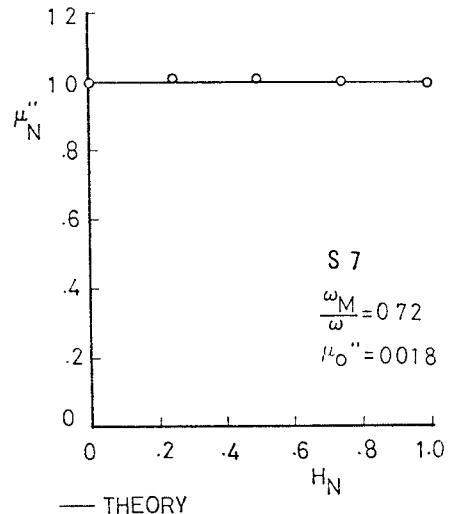


Fig. 4  $\mu_N''$  versus  $H_N$  on S7 at 9.3 GHz, where  $\mu_0'' = 0.018$ ,  $\omega_e/\omega = 0.081$  and  $b = 7^\circ$ .

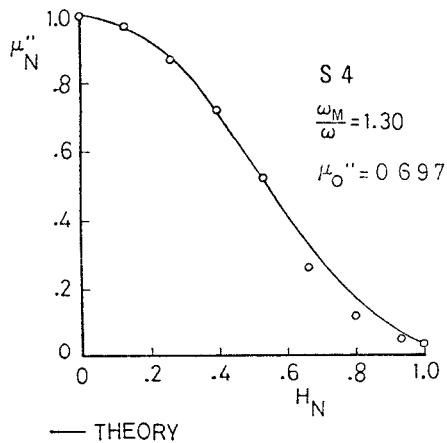


Fig. 3  $\mu_N''$  versus  $H_N$  on S4 at 3.8 GHz, where  $\mu_0'' = 0.697$ ,  $\omega_e/\omega = 0.137$  and  $b = 84^\circ$ .

SAMPLE	composition	$4\pi M_s$ [G]	$\Delta H$ [Oe]	$\omega_M/\omega$	$\mu_0''$
S3	YFe <sub>2</sub> Ca <sub>1</sub> Lu <sub>1</sub> ferrite	1400	152	1.02	0.622
S4	YIG	1780	49	1.30	0.697
S7	La <sub>2</sub> Sn <sub>3</sub> ferrite	2400	390	0.72	0.018

Table 1